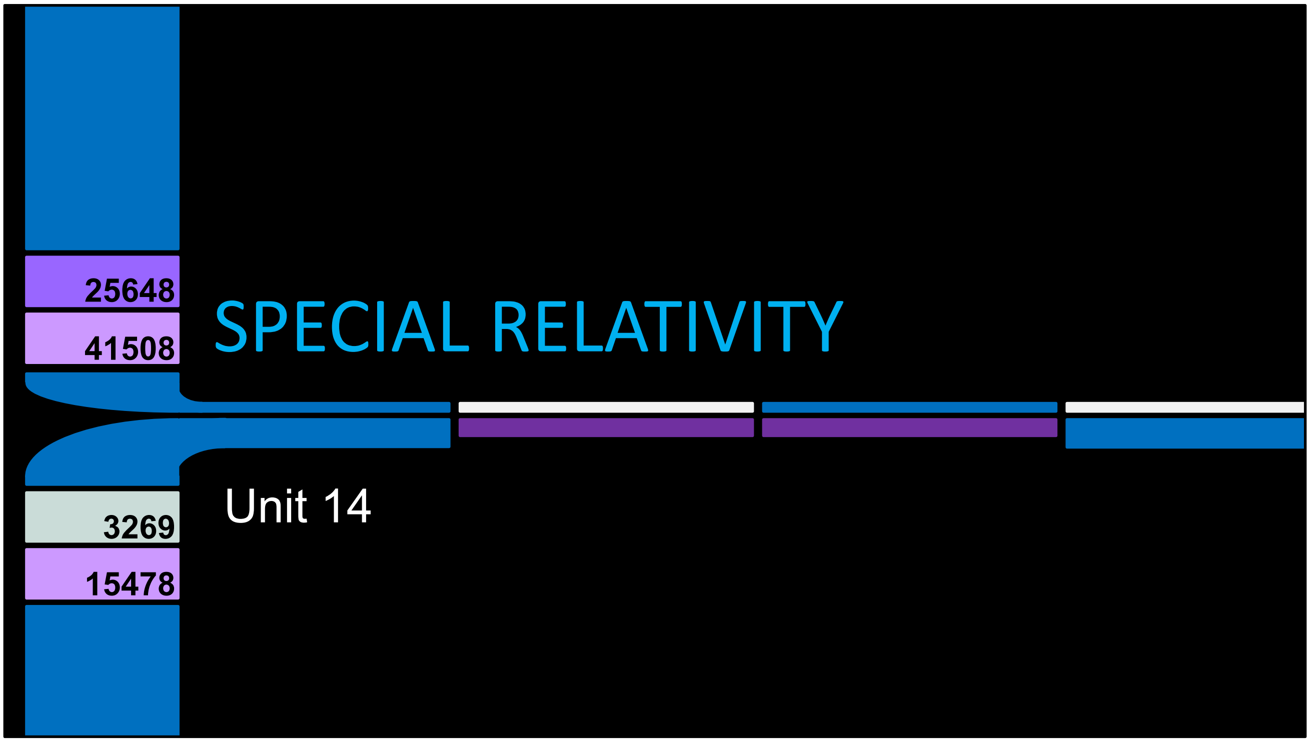

Physics Unit 12

- This Slideshow was developed to accompany the textbook
  - *OpenStax Physics*
    - *Available for free at https://openstaxcollege.org/textbooks/college-physics*
  - *By OpenStax College and Rice University*
  - *2013 edition*
- Some examples and diagrams are taken from the textbook.

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14-01 Einstein's Postulates and Time Dilation

In this lesson you will…

- State and explain both of Einstein's postulates.
- Explain what an inertial frame of reference is.
- Describe simultaneity.
- Describe time dilation.
- Calculate γ.
- Compare proper time and the observer's measured time.

14-01 Einstein’s Postulates and Time Dilation

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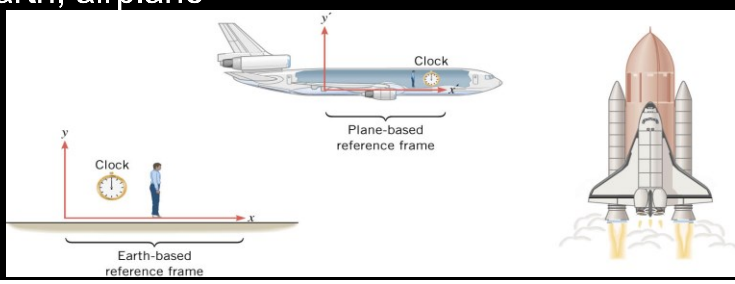
- Event
  - Physical happening in a certain place at a certain time.
- Reference Frame
  - Coordinate system (x, y, z) and clock
  - i.e. earth, airplane



Objectives: Students will correctly answer conceptual questions about the postulates of special relativity.

Students will correctly answer questions about the proper time and dilated time.

Students will correctly solve problems involving time dilation.

Focus: Einstein wondered what he would see if her were to ride a beam of light. On earth, if you travel at the same speed as a wave, the wave appears to be still relative to you. So if you were on a beam of light, you would be traveling the same speed as the wave and you would observe everything to still relative to you. That means you could see anything since no new light would come to you. Einstein didn’t like that. One day while talking with a friend about how long it takes light to arrive from various clocks in the city, Einstein solved the problem. He decided that if the speed of light was the fastest anything could travel, then all this problems were solved.

In the picture: Earth based reference frame and airplane based reference frame for observing the event of a shuttle launch

14-01 Einstein’s Postulates and Time Dilation

- Inertial Reference Frame
  - Reference frame where Newton’s Law of Inertia is valid
  - No acceleration
  - No rotation

Newton’s Law of Inertia is Newton’s First Law of Motion - Objects at rest stay at rest and objects in motion stay in motion on a straight line unless acted on by a force.

14-01 Einstein’s Postulates and Time Dilation

- Einstein built theory of special relativity on these postulates.
- The Relativity Postulate
  - The laws of physics are the same in every inertial reference frame.
- The Speed of Light Postulate
  - The speed of light in a vacuum, measured in any inertial reference frame, always has the same value of c, no matter how fast the source of light and the observer are moving relative to each other.

14-01 Einstein’s Postulates and Time Dilation

- Consequences of Relativity Postulate
  - Any inertial reference frame is as good as any other.
  - You can’t say any reference frame is truly at rest.
  - There is no absolute velocity or rest, only velocity relative to the reference frame.

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14-01 Einstein's Postulates and Time Dilation

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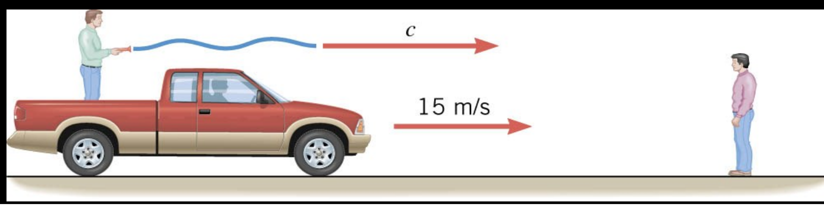
- Explanation of Speed of Light Postulate
  - The observer on the truck measures speed of light to be c since he is holding the light.
  - Logic says the observer on the ground measures the speed of light to be c + 15, but he doesn't.
  - The observer on the ground measures speed of light to be c also.
  - Verified by experiment many times.



14-01 Einstein's Postulates and Time Dilation

- Simultaneous
  - Just because two events appear simultaneous to one observer does not mean all observes see the events simultaneously

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14-01 Einstein's Postulates and Time Dilation

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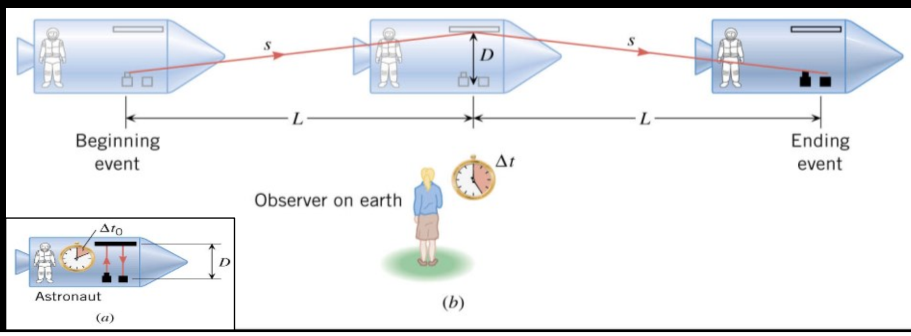
- Astronaut measures time by aiming a laser at a mirror. The light reflects from the mirror and hits a detector.
- The person on earth says that the time of the event must be longer because she sees the laser beam go farther.



14-01 Einstein's Postulates and Time Dilation

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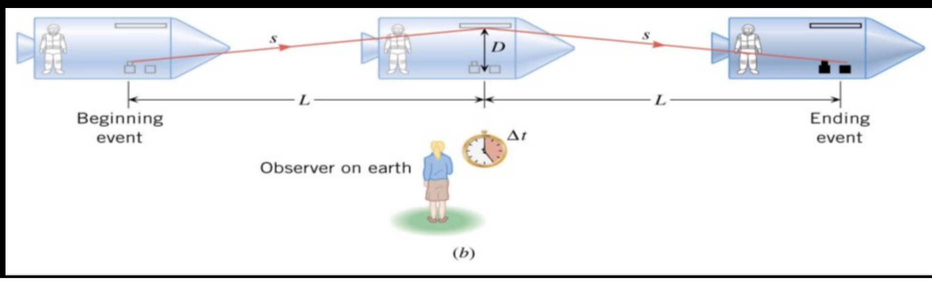
- Derivation of Time Dilation

$s = \sqrt{D^2 + L^2}$ $L = \frac{v\Delta t}{2}$ $2s = c\Delta t$

$$c\Delta t = 2\sqrt{D^2 + \left(\frac{v\Delta t}{2}\right)^2}$$



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14-01 Einstein's Postulates and Time Dilation

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- Squaring and solving for Δt gives

$$\Delta t = \frac{2D}{c}\frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- But the time the astronaut measured is

$$\Delta t_0 = \frac{2D}{c}$$

14-01 Einstein’s Postulates and Time Dilation

- Time Dilation

$$\Delta t = \gamma \Delta t_0$$

- Where
  - Δt_0 = proper time measured in a reference frame at rest relative to the event
  - Δt = dilated time measured in a reference frame moving relative to the event

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

- Where
  - v = relative speed between the observers
  - c = speed of light in a vacuum

While we don’t move at relativistic velocities (speeds that are a large fraction of c), time dilation can cause problems. For example, it would make clocks on GPS satellites out of synch with earth clocks and throw off the position.

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14-01 Einstein’s Postulates and Time Dilation

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- Let’s say the USS Enterprise’s 1/3 impulse speed is one-quarter the speed of light. If Spock, in the ship, says the planet will blow up in 10 minutes, how long does the away team have to beam up?

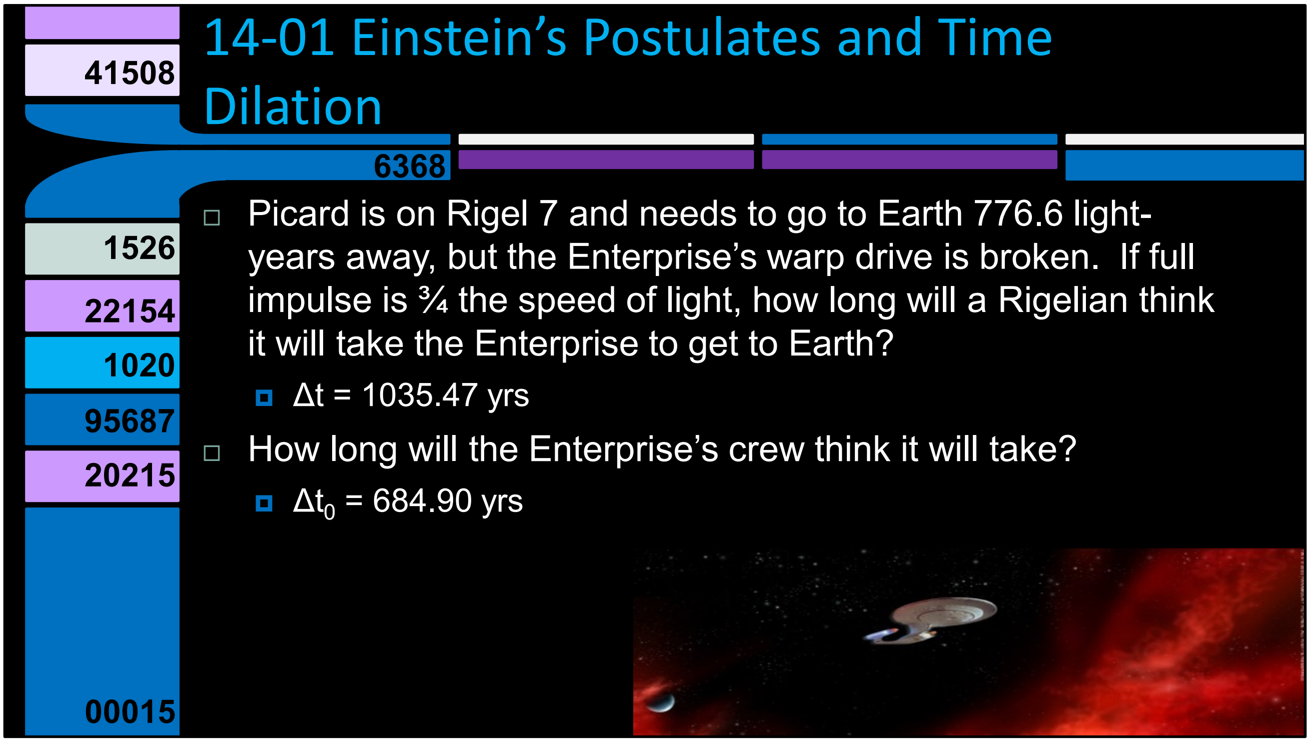
- 9.68 minutes

$$\Delta t = 10\ minutes, v = 0.25c$$

$$\Delta t = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{v^2}{c^2}\right)}}$$

$$10\ minutes = \frac{\Delta t_0}{\sqrt{1 - \left(\frac{(0.25c)^2}{c^2}\right)}} \rightarrow \Delta t_0 = 10\,\text{min}\sqrt{1 - (0.0625)} = 9.682458\ min$$

$$= 580.9\ s$$



1 light–year is the distance light will travel in a year.

a) $\Delta t = \frac{776.6\ ly}{0.75} = 1035.47\ years$ (the Rigelian measures dilated time because the events of leaving and arriving are at rest to the starship – they both happen outside the starship's windows)

b) $\Delta t = \frac{\Delta t_0}{\sqrt{1-\left(\frac{v^2}{c^2}\right)}} \rightarrow 1035.47\ yrs = \frac{\Delta t_0}{\sqrt{1-\left(\frac{(0.75c)^2}{c^2}\right)}} \rightarrow \Delta t_0 = 1035.47\ yrs\sqrt{1-0.5625} =$ $684.90\ yrs$

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14-01 Einstein's Postulates and Time Dilation

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- Time dilation was confirmed by J.C. Hafele and R.E. Keating in 1971 by taking two very accurate atomic clocks.
- One was still on earth and the other was flown around the world on commercial jets for 45 hours.
- Afterwards the clocks were compared, and the predicted difference was found.

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14-01 Practice Work

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- If you work really fast, it will seem like you took less time than an outside observer measures.
- Read 28.3

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14-02 Length Contraction

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In this lesson you will…
- Describe proper length.
- Calculate length contraction.
- Explain why we don't notice these effects at everyday scales.

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14-02 Length Contraction

- Since the observer moving with the event measures a different time than the observer not moving with the event, are the lengths different?

Objectives: Students will correctly distinguish between contracted length and proper length.

Students will correctly solve problems involving length contraction.

Focus: Answer questions on previous assignment. See this slide. (Since the observer moving with the event measures a different time than the observer not moving with the event, are the lengths different?)

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14-02 Length Contraction

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- $x = vt$
- Both observers agree on v
- t is different by $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$
- So x must be different by $\frac{1}{\sqrt{1-\frac{v^2}{c^2}}}$ also

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14-02 Length Contraction

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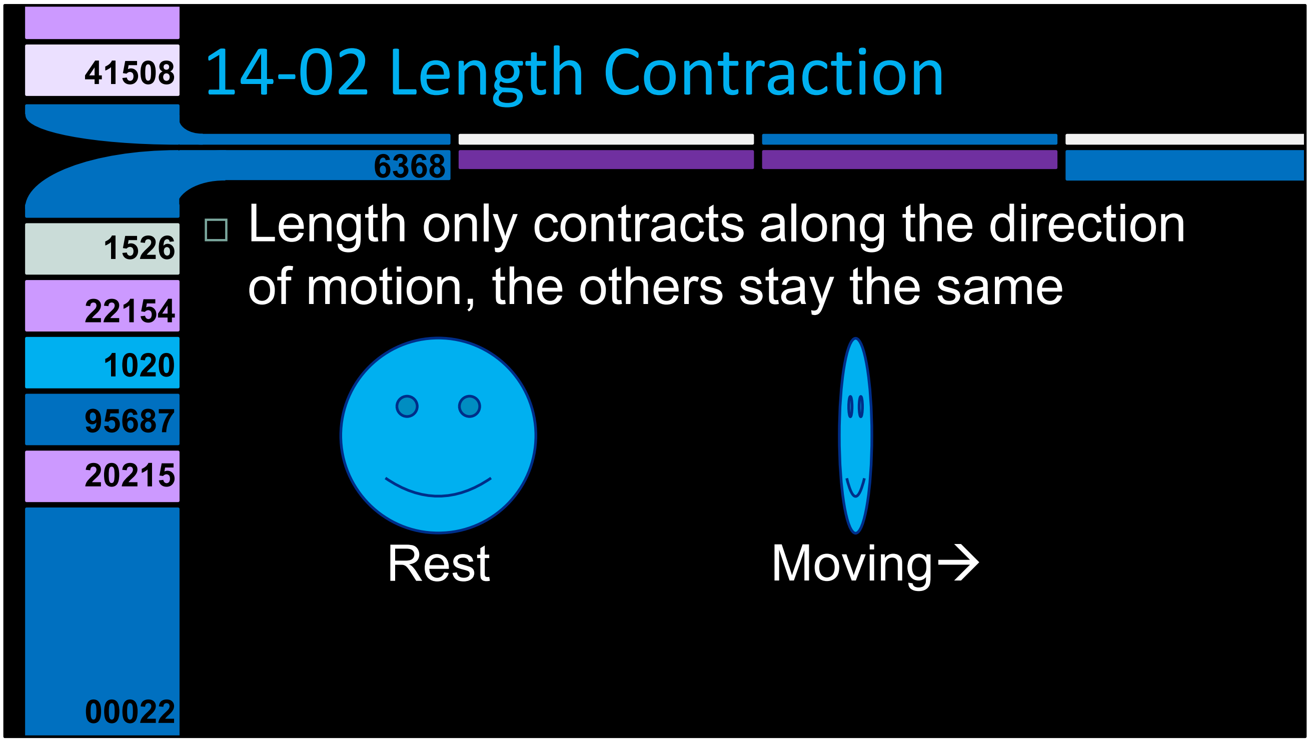
- The distance measured by a person at rest with the event is shorter than that measured by person at rest with respect to the endpoints.
- $L = L_0\sqrt{1-\frac{v^2}{c^2}} = \frac{L_0}{\gamma}$
- L_0 = proper length
  - Length between 2 points as measured by person at rest with the points.



If you moved near the speed of light you would be the same height, but very thin.

This is as observed by the person moving with the event.

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14-02 Length Contraction

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- When the Starship Enterprise travels at impulse (v = 0.7c), a ground-based observer measures the ship as 707 ft long. How long does the crew measure the ship?

- 990 ft

$$707\ ft = L_0\sqrt{1-\frac{(0.7c)^2}{c^2}}$$
$$L_0 = 990\ ft$$

Of course all the measuring devices are shortened as well, so the measurements would be the same.

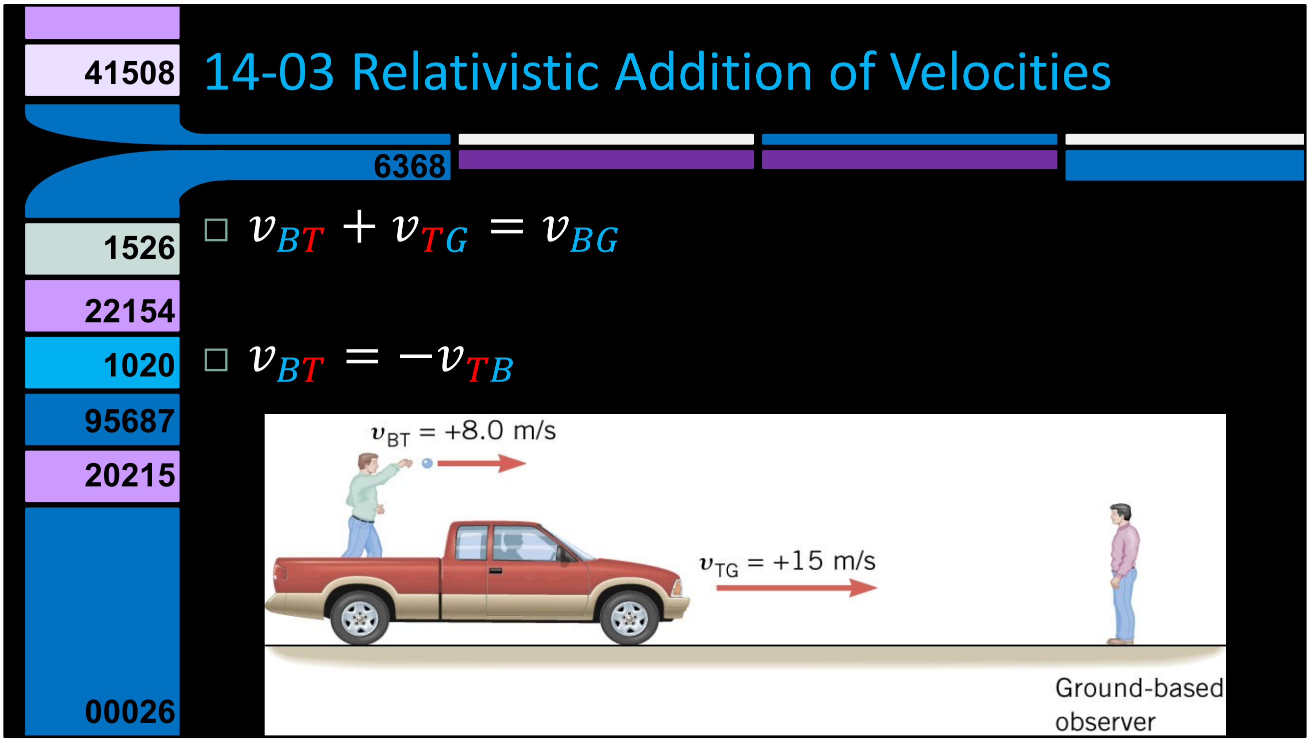
14-02 Pratice Work

- Don’t stretch these problems out too long.
- Read 28.4

14-03 Relativistic Addition of Velocities

In this lesson you will…

- Calculate relativistic velocity addition.
- Explain when relativistic velocity addition should be used instead of classical addition of velocities.
- Calculate relativistic Doppler shift.



Objectives: Students will correctly distinguish between when relativistic addition of velocities is used and when nonrelativistic addition is used.

Students will correctly solve problems involving relativistic addition of velocities.

Students will correctly solve problems involving nonrelativistic addition of velocities.

Focus: Answer questions on previous assignment. Review nonrelativistic addition of velocities.

Velocity of Ball to Truck + Velocity of Truck to Ground = Velocity of Ball to Ground
Subscripts show the order

14-03 Relativistic Addition of Velocities

- What if the combination of the truck and the ball added to be more than the speed of light?
- The ground-based observer would observe the ball to travel faster than light.
- This cannot happen.

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14-03 Relativistic Addition of Velocities

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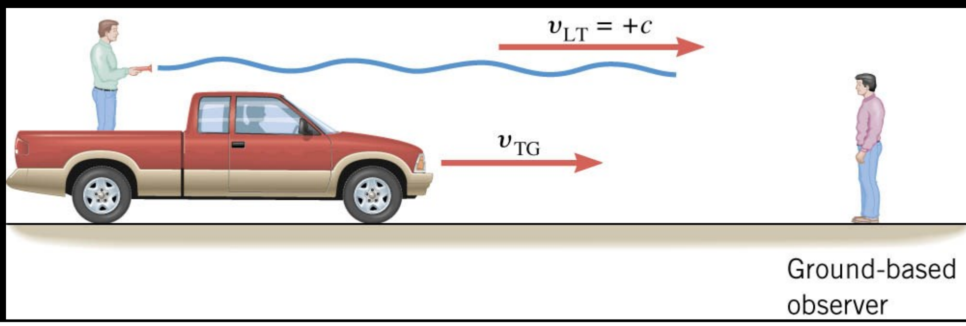
- Relativistic Addition of Velocity

$$v_{LG} = \frac{v_{LT} + v_{TG}}{1 + \frac{v_{LT} v_{TG}}{c^2}}$$



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14-03 Relativistic Addition of Velocities

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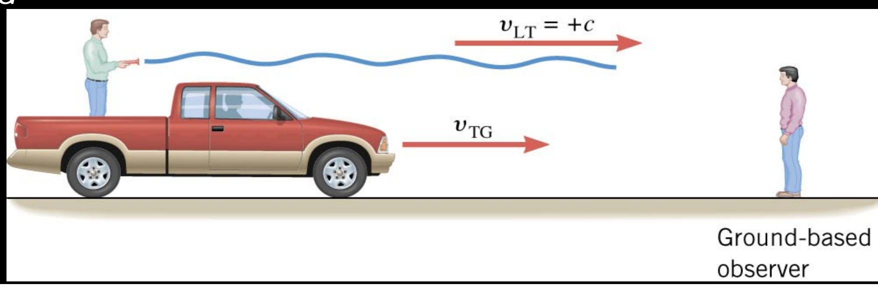
- At what speed does the ground based observer see the light travel?

- $v_{LG} = c$



$$v_{LG} = \frac{v_{LT} + v_{TG}}{1 + \frac{v_{LT} v_{TG}}{c^2}} = \frac{c + v_{TG}}{1 + \frac{c v_{TG}}{c^2}} = \frac{c + v_{TG}}{1 + \frac{v_{TG}}{c}} = \frac{c + v_{TG}}{\frac{c}{c} + \frac{v_{TG}}{c}} = \frac{c + v_{TG}}{\frac{c + v_{TG}}{c}} = \frac{(c + v_{TG})c}{c + v_{TG}} = c$$

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14-03 Relativistic Addition of Velocities

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- Doppler shift for relative velocity

$$\lambda_{obs} = \lambda_s \sqrt{\frac{1 + \frac{u}{c}}{1 - \frac{u}{c}}}$$

$$f_{obs} = f_s \sqrt{\frac{1 - \frac{u}{c}}{1 + \frac{u}{c}}}$$

- *u* is relative velocity of source to observer
  - *Positive if moving away*

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14-03 Relativistic Addition of Velocities

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- The starship *Enterprise* moves at 0.9c relative to the earth and a Klingon Bird-of-Prey moves the same directions at 0.7c relative to the earth. What does the navigator of the Bird-of-Prey report for the speed of the *Enterprise*?
- $v_{EntK} = 0.541c$

$$v_{EntE} = 0.9c, v_{KE} = 0.7c$$

$$v_{EntK} = \frac{v_{EntE} + v_{EK}}{1 + \frac{v_{EntE} v_{EK}}{c^2}}$$

$$v_{EntK} = \frac{(0.9c \pm 0.7c)}{1 + \frac{(0.9c)(-0.7c)}{c^2}}$$

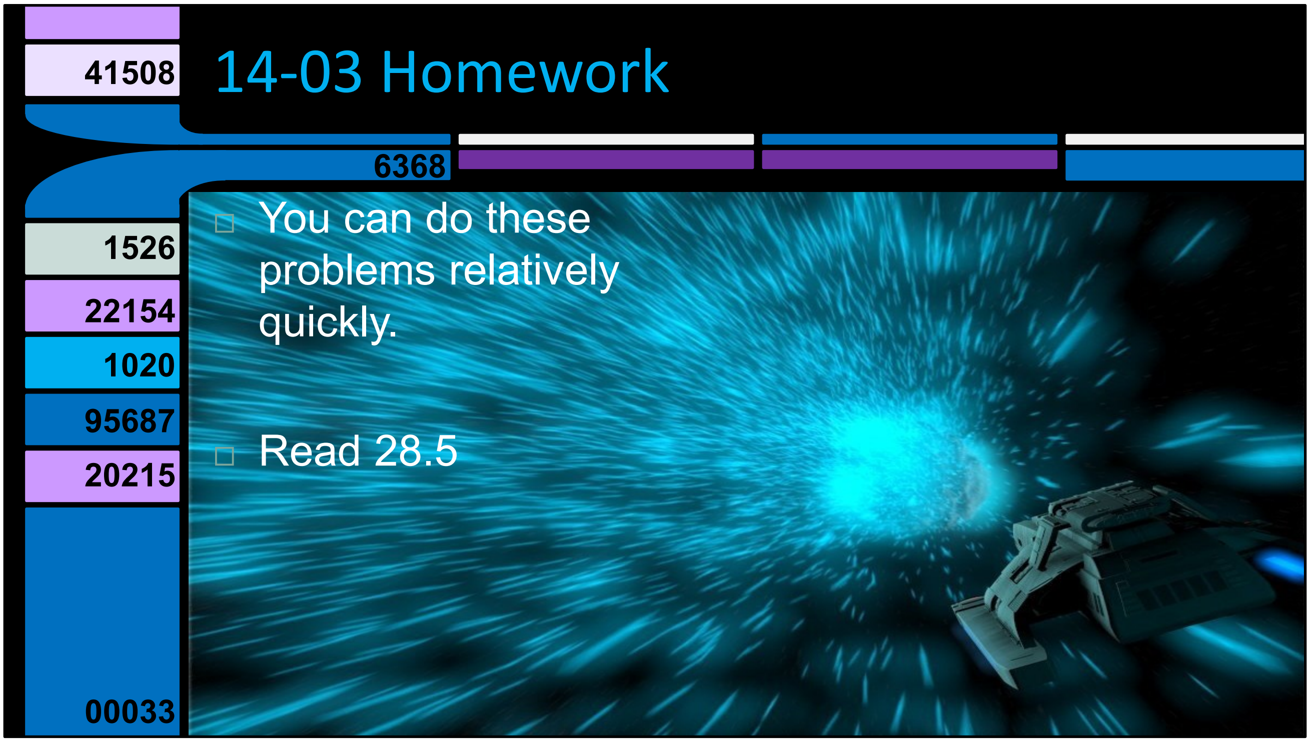
$$v_{EntK} = 0.541c$$

14-03 Relativistic Addition of Velocities

- The starship *Enterprise* moves at 0.9c relative to the earth and a Klingon Bird-of-Prey moves the same directions at 0.7c relative to the earth. If the *Enterprise* has blue (λ = 475 nm) lights, what wavelength does the Klingon ship see as it leaves?
- $\lambda_{obs} = 870\ nm$, infrared

$$\lambda_{obs} = \lambda_s \sqrt{\frac{1+\frac{u}{c}}{1-\frac{u}{c}}}$$

$$\lambda_{obs} = (475\ nm)\sqrt{\frac{1+\frac{0.541c}{c}}{1-\frac{0.541c}{c}}} = 870\ nm$$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14-03 Homework
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You can do these problems relatively quickly.
Read 28.5

14-04 Relativistic Momentum

In this lesson you will…

- Calculate relativistic momentum.
- Explain why the only mass it makes sense to talk about is rest mass.

14-04 Relativistic Momentum

- Law of Conservation of Momentum
  - The total momentum of a closed system does not change.
  - p = mv
  - However, when v approaches c, we must adjust the formula
  - $$p = \frac{mv}{\sqrt{1-\frac{v^2}{c^2}}}$$

Objectives: Students will correctly solve problems involving relativistic momentum.
Focus: Answer questions on previous assignment. Review what the law of conservation of momentum is.

14-04 Relativistic Momentum

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- Relativistic momentum is always higher than nonrelativistic momentum because
- $\sqrt{1 - \frac{v^2}{c^2}} < 1$
- Since we divide by the radical in the formula, the result is a larger number.

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14-04 Relativistic Momentum

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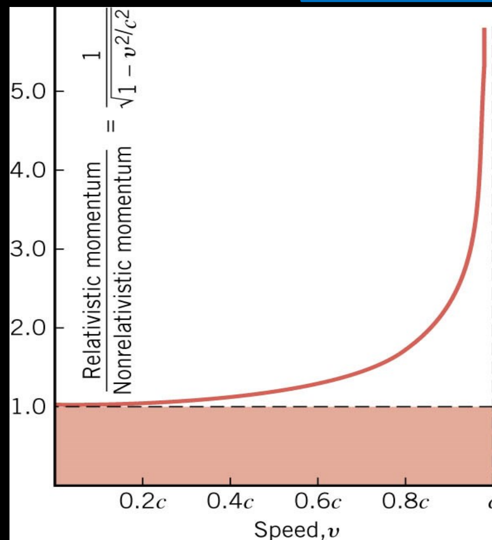
- Notice that when the speed is near 0, the relativistic momentum is near the nonrelativistic.
- When the speed is near c, the relativistic momentum increases exponentially.



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14-04 Relativistic Momentum

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- In a game of Dom'Jot, a small ball (0.5 kg) is hit across a table. If the ball moving at 3 m/s and the speed of light in a vacuum is 4 m/s, what is the relativistic momentum of the ball?
  - p = 2.27 kg m/s
- The nonrelativistic momentum?
  - p = 1.5 kg m/s

$$p = \frac{0.5\ kg \cdot 3\frac{m}{s}}{\sqrt{1 - \frac{\left(3\frac{m}{s}\right)^2}{\left(4\frac{m}{s}\right)^2}}} = 2.27\ kg\frac{m}{s}$$

$$p = \left(0.5\ kg \cdot 3\frac{m}{s}\right) = 1.5\ kg\frac{m}{s}$$

Dom'Jot is a game is Star Trek similar to pool or billards.

14-04 Homework

- Build some momentum as you attempt these problems.
- Read 28.6

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14-05 Relativistic Energy

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In this lesson you will…

- Compute total energy of a relativistic object.
- Compute the kinetic energy of a relativistic object.
- Describe rest energy, and explain how it can be converted to other forms.
- Explain why massive particles cannot reach *c*.

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14-05 Relativistic Energy

- The total energy of an object
- $E = \frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}}$
- If the object is not moving, the ***rest energy*** is
- $E_0 = mc^2$

Objectives: Students will correctly solve problems involving energy of objects.
Students will correctly solve problems involving kinetic energy at relativistic speeds.
Students will correctly answer conceptual problems about the mass-energy relationship.
Focus: Answer questions about previous assignment. Everybody knows that Einstein discovered that $E = mc^2$ but there is more to it.

Discovered by Einstein

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14-05 Relativistic Energy

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- How much energy is in a 5-gram pen at rest?
  - $E_0 = 4.5 \times 10^{14} J$
- How long will that run a 60-W light bulb?
  - $t = 7.5 \times 10^{12}\ s$
  - $= 237665\ yr\ 9\ months$

$$E = (0.005\ kg)\left(3.00 \times 10^{8} \frac{m}{s}\right)^{2} = 4.5 \times 10^{14}\ J$$

$$P = \frac{W}{t} \rightarrow 60\ W = 4.5 \times 10^{14}\ \frac{J}{t} \rightarrow t = 7.5 \times 10^{12}\ s = 237665\ yr\ 9\ months$$

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14-05 Relativistic Energy

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- If the object is moving, then the total energy is $E = E_0 + KE$
- $\frac{mc^2}{\sqrt{1-\frac{v^2}{c^2}}} = mc^2 + KE$
- $KE = mc^2\left(\frac{1}{\sqrt{1-\frac{v^2}{c^2}}} - 1\right)$

KE = kinetic energy

At speeds far less than c, this becomes about $\frac{1}{2}mv^2$

14-05 Relativistic Energy

- Mass and energy are the same
- A change in one, means a change in the other.
- For example, you pick up your backpack and increase its gravitational potential energy.
  - Since the energy increases, the mass must increase.
  - So when you carry your backpack, it is actually heavier than when it is on the ground.

Yes, the change in mass is incredibly small.

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14-05 Relativistic Energy

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- The sun radiates electromagnetic energy at 3.92×10^{26} W. How much mass does the sun lose in 1 year?
  - 1.37×10^{17} kg
- This is only a tiny fraction of the sun's mass (6.9×10^{-14})

$$P = \frac{W}{t} \to 3.92 \times 10^{26}\ W = \frac{E}{31557600\ s} \to E = 1.237 \times 10^{34}\ J$$

$$E = mc^2 \to 1.237 \times 10^{34}\ J = m\left(3.00 \times 10^{8}\ \frac{m}{s}\right)^2 \to m = 1.37 \times 10^{17}\ kg$$

14-05 Relativistic Energy

- A final consequence
- Objects with mass cannot reach the speed of light.
- This is because it would take an infinite amount of energy.

The denominator of the KE formula would go to zero, which means value of the fraction goes to infinity.

14-05 Homework

- You have less mass if you use energy.